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Financing an Uncertain Retirement Part I: Spending Strategies

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Executive Summary

- Real-world retirees exhibit several behaviors that conflict with the predictions of canonical retirement income models. Often, this is presented as evidence of irrational behavior on the part of retirees.
- This paper, the first in a two-part series, takes a different approach. A key component of retirement is uncertainty: We do not know how long assets must last or even how much we will have to spend.
- Uncertain future spending needs dramatically alter the results of typical retirement income models and generate predictions that more closely reflect real-world behaviors.

INTRODUCTION

The literature on the savings and spending behavior of retirees spans academia, financial advisors and industry professionals. Often, the real-world behavior of retirees is in conflict with what would be predicted by rational agents behaving according to economic models of retirement income.

These models typically begin with savings and investment decisions defined over the retiree's remaining life cycle: Much like the financial industry, these models focus on how to manage an individual's assets. At the same time, there is an equally large literature in economics describing how real-world spending, savings and portfolio allocation outcomes are heavily influenced by realized liabilities in retirement. This paper marries these two approaches in a single analytical framework. The inclusion of uncertain, unhedgeable liabilities helps illuminate several otherwise irrational retiree behaviors.

The principal contribution of this paper is to propose, parameterize and solve a life-cycle decumulation problem with uncertain liabilities. Uncertain future spending alters a household's problem from one with even, predictable needs to one in which there is great (and likely increasing) uncertainty about both how much money will be needed later in life and exactly when it will be necessary. The results help address several empirical puzzles in retiree behavior.

In the first section, a typical asset-focused retirement income model is presented and solved. Several conclusions of this model are at odds with real-world behaviors. Although there are many such puzzles, this paper focuses on wealth drawdown rates that are too slow, consumption that is too volatile and annuitization rates that are too low. The second section reviews the literature that shows a plausible connection between uncertain spending needs in retirement and real-world savings and investment decisions. Finally, the canonical retirement income model is augmented with a realistic parameterization of uncertain liabilities and is solved.

1. A TYPICAL MODEL

In this model, a retirement-age investor chooses how much of their initial wealth W_0 to consume in each period of their retirement, with preferences over consumption in period t given by $U(c_t)$, a rate of preference discounting $\beta < 1$ and a survival rate given by $\pi_t < 1$. Formally, the investor solves

$$\max_{\{c_s\}_{s=0}^{T}} \sum_t \beta^t \pi_t U(c_t).$$

Subject to the budget constraint that their total consumption must be affordable given their initial wealth,

$$\sum_t c_t = W_0.$$

This leads to the well-known Euler equation relating consumption between periods:

$$U'(c_t) = \beta \pi_t U'(c_{t+1}).$$

For example, constant relative risk aversion (CRRA) preferences $\left(U(c_t) = \frac{c_t^{1-\rho}}{1-\rho}\right)$ yields an optimal consumption profile that declines $(\beta\pi_t < 1)$ according to the following relationship:

$$\frac{c_{t+1}}{c_t} = (\beta\pi_t)^{\frac{1}{\rho}}.$$

When this Euler equation is combined with the budget constraint, the optimal amount of wealth allocated to consumption in each period is uniquely identified. The spending problem here is distinct from an asset allocation problem. It is possible to say a great deal without considering asset markets. While this is an obvious simplification, such an approach has the advantage that the conclusions and interpretation of the results do not depend on assumptions about the future of the capital markets.

Typical implementations of this model include uncertain rates of return, uncertain investment or inflation outcomes and choice over asset allocation. Though the details may differ, a model such as this is an appealing characterization of the problem the retiree faces. These sorts of models have formed the foundation of much retirement income research since their introduction over 50 years ago in Yaari (1965). This ubiquity is somewhat surprising. There are a number of implications of this framework that do not map to actual observed behavior.

Puzzle 1: Low decumulation rates

The first issue concerns the rate that assets are spent down. The model above implies that retirees should draw down their assets much faster than is seen in the data. Even if we attempt to slow down the model predictions with conservative assumptions on life expectancy and discounting, it is nearly impossible to reconcile the model with actual behavior without adding to its structure.

Initially, we consider three hypothetical cases (see Exhibit 1). Case 1 uses survival rates for a 65-year-old male, with a rate of time preference of 2.5% and a coefficient of relative risk aversion of 4. Case 2 is identical, but for a 65-year-old female (with longer life expectancy). Finally, Case 3 is an upper bound of what is possible with parameters alone: It represents an individual who will live to age 105 with 100% probability and has no rate of time preference.

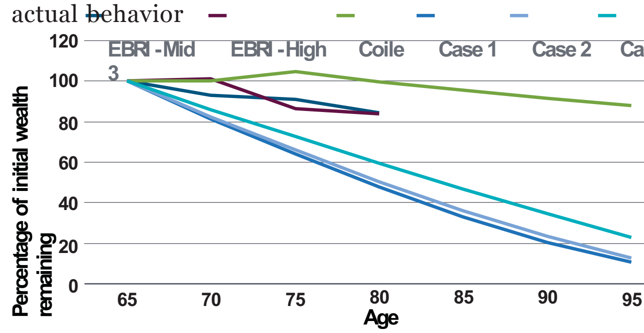
Exhibit 1: Preference parameters

Variable	Symbol	1	2	3
Survival rates	π	Male	Female	105
Discounting	β	2.5%	2.5%	0.0%
Risk aversion	ρ	4	4	4

Hypothetical example for illustrative purposes only.
Source: Aura

Exhibit 2 depicts the model-implied behavior for the three parameterizations above, alongside actual decumulation rates as documented by the Employee Benefit Research Institute (EBRI) and Coile and Milligan (2009). All of these specifications, even the unrealistically long-lived, very patient version, would decumulate wealth throughout retirement much faster than

Exhibit 2: Model-implied wealth decumulation rates versus actual behavior



Hypothetical example for illustrative purposes only.
Source: Author's calculations, EBRI (2018), Coile and Milligan (2009)

n addition to their analytical convenience, CRRA preferences are plausible in a retirement context given the focus on replacement rates when retirement income is discussed. A replacement rate target that is independent of wealth or other parameters suggests a relative risk aversion that is constant.

actual retirees would. The model predicts that by age 75 a retiree would have spent down to 65%–75% of their assets, while real-world retirees still hold 90%-plus of their initial balance. (In fact, Coile and Milligan documents a slight increase in asset balances after age 65.)

In short, it is not possible to bridge the gap between model predictions and actual behavior with different parameters.

There are many potential behavioral explanations for this phenomenon. Some show promise, though it is generally difficult to reconcile a far-sighted reluctance to spend in retirement with the well-known myopic unwillingness to save while working. Many in the retirement industry simply force their models to have slower-than-optimal spending rates. This constrains the problem but does not explain it. Instead, with modest, realistic additions to this framework, it is possible to produce much slower decumulation rates, closer to what is seen in real-world data, without appealing to irrational behavior on the part of retirees. First, though, let us describe another puzzle.

Puzzles 2 and 3: Volatile consumption and lack of annuity purchases

Retirees in Yaari-style models smooth out their consumption. Any unexpected shocks to wealth manifest evenly in consumption over the remaining lifetime as near-even shifts from previous levels. In the world described by the model above, spending volatility is suboptimal. Retirees should strongly favor a steady replacement rate, and their primary risk is outliving their assets (often referred to as "longevity risk"). An annuity, which provides a fixed payment for as long as the individual is alive, is a well-paired product for such a framework. In general, the actuarial rate of return on an annuity – for the surviving annuitants, anyway – will eventually dwarf any realistic asset market assumptions.

Needless to say, zero consumption volatility and heavy use of annuities are not what actual retirement behavior looks like. Annual volatility in retirement spending ranges from 5%–10% of total spending, depending on age. In 2019, J.P. Morgan Asset Management research found that 80% of retirees experienced "substantial" changes in spending, with nearly one quarter spending either 20% more or 20% less from year to year. Despite annuities' relatively limited use, their theoretical appeal has been a staple of the literature since their inception: Yaari (1965) showed that life-cycle consumers should annuitize all of their savings under certain conditions (such as those in the baseline model above). Brown, Davidoff and Diamond (2005) went significantly further and proved that annuitization is the dominant strategy under significantly more relaxed circumstances, and highlighted the theoretical appeal of state-contingent annuity products. In these models, the estimates of welfare gains from the deferred annuities are as high as 20% (Horneff, Maurer and Mitchell 2016), and most models find that essentially all retirees would benefit from some allocation to a deferred annuity. The actual annuity market, however, offers almost a perfect contrast to this theoretical appeal. The life annuities market in the U.S. is vanishingly small – so much so that simply collecting enough data for empirical work is difficult (Brown 2009).

The model described above is no different. To see this, consider a modest extension in which the model allows for the purchase of an immediate annuity before the first period that provides a fixed payment for the remainder of the retiree's life. To evaluate the impact of pre-existing annuities, we will also consider a case in which the retiree receives Social Security benefits.

Altogether, the objective function changes to

$$\max_{\{c_s\}_{s=0}^{T}} \sum_t \beta^t \pi_t U(c_t + Ann + SS) \text{ s.t.}$$

$$\sum_t c_t + A_0 = W_0$$

$$Ann = \frac{A_0}{P}$$

where SS is the Social Security payment, Ann is the amount of the annuity payment, A_0 is the amount of initial wealth invested in the annuity and P is the annuity's price. For simplicity, we set the Social Security payment equal to the statutory maximum for an individual claiming benefits at their full retirement age ($34,300 per year in 2019). The annuity is priced above the actuarially fair value to reflect actual market prices for immediate nominal annuities. Even when annuities are "expensive," however, an agent in this model heavily favors annuitization.

2 Annuities are subject to the claims-paying ability of the issuing insurance company. Aura does not offer insurance guaranteed products or products that offer investments containing both securities and insurance features.

3 Fellowes (2017)

4 Roy, Kim-Steiner (2019)

5 They also discuss circumstances in which annuities may be less attractive. In the next section, we will explore one of these.

6 This is usually referred to by the inverse: The "money's worth" of an annuity is the ratio of net present value of expected payouts to the price. Estimates in the literature show money's worth values of approximately 85% (Brown, Mitchell and Poterba 2000). The baseline case here is slightly more expensive (80%), and annuities are still heavily favored.

Exhibit 3: Optimal annuitization rates in the canonical framework

	Life expectancy	Risk aversion	Discounting	Annuity share	With Social Security
1	Male	4	2.50%	89%	100%
2	Female	4	2.50%	86%	100%
3	105	4	0.00%	100%	100%
4	Male	8	2.50%	100%	100%
5	Male	2	2.50%	81%	65%

Hypothetical example for illustrative purposes only. The model outputs included here are not based on any particularized financial situation, or need, and nothing contained herein should be considered investment or retirement advice. Aura does not offer insurance guaranteed products or products that offer investments containing both securities and insurance features. The model is limited to analyzing the optimal annuitization rates. Investors should speak to their financial professional regarding the investment mix that may be right for them based on their financial situation and investment objectives.
Source: Author's calculations

Life expectancy data is from the Social Security Administration. Annuities are priced at a money's worth of 80%. Life expectancy of 105 represents a retiree who lives to age 105 with 100% probability.

Our baseline parameterization is a 65-year-old male with a relative risk aversion of 4 and a rate of time preference of 2.5%. Despite a money's worth of 80%, this model suggests that nearly 90% of his initial wealth should be invested in annuities. Similarly, a 65-year-old female with the same preference parameters would invest 86% of her assets. For more (less) risk-averse households, this ratio increases (decreases) further, but no set of preference parameters short of very high mortality risk would recover anything near the annuity purchase rates that we see in the data.

There are several potential competing explanations for the lack of a thriving annuity market, including endowments of pre-existing annuities, such as Social Security. The existence of Social Security appears unable to address the issue. When we include Social Security payments (see Exhibit 3), optimal annuitization rates tend to increase, except for individuals with lower risk aversion, who are willing to shift their consumption over time toward the periods when they are likely to be alive, rather than potentially receiving increased payments later in life, conditional on survival.

Perhaps the most cited reference to the annuitization puzzle is from Franco Modigliani's 1985 Nobel acceptance speech: "It is a well-known fact that annuity contracts, other than in the form of group insurance through pension systems, are extremely rare. Why this should be so is a subject of considerable current interest. It is still ill-understood." Since that time, there have been several attempts to explain this difference, appealing to both rational and behavioral motives with some success.[7] The limited real-world appeal of annuities, given their obvious strengths in canonical models, remains an open question.

2. CLOSING THE GAP: AN UNCERTAIN RETIREMENT

More than 60% of Americans report that they don't know how much money they will need to retire.[8] Guided by the conclusions of models similar to those described above, financial advisors and asset managers have focused on how to manage retiree assets, targeting an income replacement requirement. That is, retiree concerns around "running out of money" are often interpreted as uncertainty about their assets: how to invest, how to draw down their wealth or how much income can be financed from existing balances at different ages. The contrast between the model-implied solutions to these sorts of questions and actual observed behavior leads to the puzzles described in the previous section.

Instead of focusing on the assets, here the model is written to focus on uncertainty in the retiree's liabilities. Comments such as "I don't know how much money I will need in my retirement" are interpreted literally: The retiree faces required future expenses, but their size and timing are unknown. Sources of these shocks could include unexpected home repairs or maintenance, potential tax/policy changes or even as a problem-solving heuristic in the presence of uncertain future preferences. While the applications here are broad, this exposition focuses on a single, measurable and large source of uncertainty: healthcare expenses.

Healthcare costs are a significant concern. Only 36% of individuals are confident that they can cover their healthcare costs in retirement. This concern is not unfounded: Even after accounting for Medicare, out-of-pocket healthcare expenses are large and volatile. Once individuals reach age 70, their households will, on average, incur over $122,000 in *out-of-pocket* medical spending over the remainder of their lives, while

7 Brown et al. (2008) and Beshears et al. (2014) are particularly compelling.

8 www.aura.co.th 2018)

9 These shocks must not be recreational; barring some requirement for the expense, they would not be consumed. (The marginal utility of the expense in a state where it is not required is zero.) This may not be an accurate description for routine or minor medical expenses, but it is certainly plausible for the large and volatile expenses that drive the bulk of the uncertainty. That is, this framework requires that retirees do not purchase unnecessary, recreational cancer treatments or emergency surgeries.

10 T. Rowe Price (2017)

households in the highest percentile will see over $600,000 in medical spending. Though these shocks occur strictly outside of asset markets, they affect saving and spending behavior.

There is substantial literature detailing the relationship between healthcare expenses and savings and the portfolio-choice behavior of the elderly. De Nardi, French and Jones (2010) show that medical expenses are a key driver of low decumulation rates. They find that in the absence of medical expenses the rate of drawdown of wealth would more than double between the ages of 74 and 84, an effect that dominates the magnitudes of other common explanations, such as bequest motives. Coile and Milligan (2009) document significant wealth and asset allocation consequences of health expenses in retirement, including the sale of stocks and drawdown of IRAs, as well as a reduction in primary housing assets after health shocks. Rosen and Wu (2004) show that in addition to lower overall levels of assets, sick households hold a higher fraction of their portfolios in low-risk assets than do healthy households; particularly relevant for investment advisors, this effect appears to be twice as large as the retirees' overall attitudes toward risk. Similarly, Poterba, Venti and Wise (2017) estimate that from 1992–2008 households with "excellent" self-reported health experience accumulated nearly 50% more assets than those with "poor" health did – equivalent to an extra 250 basis points (bps) of asset returns when annualized. In their 2011 paper, Poterba, Venti and Wise note similarly large effects and, like Coile and Milligan, show that people deploy their home equity and nonannuitized wealth as a precautionary reserve for very long-life or substantial medical outlays. The consequences of healthcare shocks on wealth and portfolio choice in retirement are large, especially when compared with the impact of typical risk and asset allocation modifications advocated in the industry.

Healthcare costs are obviously important, though they are only one example of uncertain or unanticipated future expenses. Given the array of potential risks and uncertainties, it is not economical (or even feasible) to purchase complete insurance for every eventuality. However, readily available data on the magnitude of these expenses allows us to explore this question. Unsurprisingly, both the mean and the variance of healthcare expenses increase with age, and the distribution of health expenses has a very large right tail. For simplicity, we assume annual required expenses are independently distributed, and follow distributions based on the data on

11 Jones et al. (2018)

out-of-pocket medical expenses by age reported in Jones et al. (2018). Exhibit 4 shows the resulting distribution of required healthcare expenses.

Exhibit 4: Parameterized out-of-pocket healthcare expenses ($000)

Age	65	75	85	95	Lifetime
μ	0.41	0.98	1.37	1.65	
σ	1.34	1.39	1.46	1.51	
Mean	3.1	5.9	9.3	13.2	126
Median	1.5	2.7	3.9	5.2	116
75th percentile	3.6	6.7	10.3	14.2	150
90th percentile	7.8	14.9	23.9	34.0	193
95th percentile	12.1	23.5	38.5	55.8	222

Hypothetical example for illustrative purposes only. Source: Author's calculations

Values are in thousands. Lifetime values reflect the Social Security Administration's published survival rates for a 65-year-old male. The simulation is drawn from a lognormal distribution parameterized by mean μ and variance σ^2.

Once it is weighted to reflect survival rates, this process recovers a lifetime distribution of out-of-pocket costs similar to what is observed in the data (Jones et al. 2018 and EBRI 2014), with the top percentiles exceeding $200,000 in present value. With a median net worth at retirement of approximately $250,000 in 2016, according to the Survey of Consumer Finances, it is no wonder that so many retirees fear they may not be able to afford their healthcare expenses in retirement.

3. A MODEL OF UNCERTAIN CONSUMPTION REQUIREMENTS

We now extend the model in Section 1so that preferences are defined over expected utility, itself a function of consumption in excess of some unknown minimum level h_t:

$$\max_{\{c_s\}_{s=0}^{T}} \sum_t \beta_t \pi_t E[U(c_t - h_t)] \text{ subject to}$$

$$c_t \le W_t + SS + Ann_t - h_t.$$

If the required consumption exceeds available funds that period, then all assets are depleted and consumption is fixed at a guaranteed minimum level c_t for all remaining periods:

12 These shocks are parameterized in absolute dollars, though there is evidence that healthcare expenses increase with income levels. More complex health cost models are common, such as Markov transitions, autocorrelated shocks or endogenous mortality later in life, but for a healthy new retiree, as in this model, planning with an unconditional risk distribution is substantively similar to a conditionally autocorrelated process. Finally, this analysis does not account for the fact that healthcare prices typically rise faster than inflation/ discretionary consumption; this would exacerbate the ultimate effects.

$$\text{If } h_t > W_t + SS + Ann_t, \ \{c_{t+s} - h_{t+s}\}_{s=0}^{T} = \{c_{t+s}\}_{s=0}^{T}, \ \{W_{t+s}\}_{s=0}^{T} = 0.$$

This seems realistic. Individuals receive Medicaid coverage with very small out-of-pocket costs only after paying a deductible of essentially all their remaining assets.[13] Of course, with any type of guarantee, it is possible for a rapid divesting of all assets to be the dominant strategy, as this maximizes the value of the guarantee for the remaining years. This is not a widely observed strategy among retirees (and does not reflect typical guidance from financial professionals), suggesting that real-world welfare guarantees represent a rather low level of consumption. Consistent with this, we select a guarantee equal to the fifth percentile of income for individuals in the U.S. aged 65 and over.[14] Reasonable changes in the level of the guarantee will impact the magnitude of the results in this paper but will not affect their existence.[15]

With uncertain expenses, wealth decumulation rates become state dependent. Optimal consumption in each year (c_t) is given by a function of age, current wealth levels (W_t) and current minimum spending requirements (h_t): $c^*(t, W_t, h_t)$.. Given the process for spending requirements (h_t) in Exhibit 4, it is possible to numerically solve for the optimal consumption function through recursion.

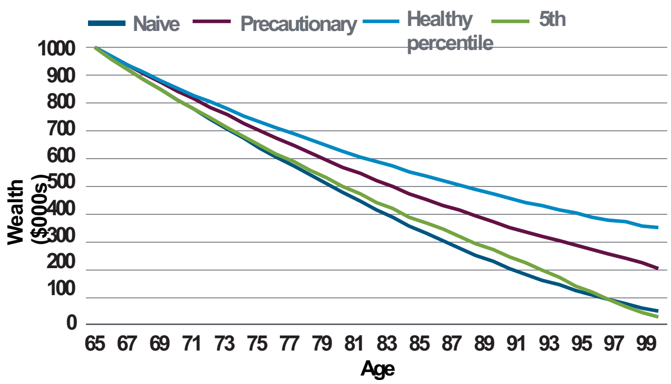
This addition to the model leads to substantively different behavioral predictions, particularly for the three puzzles discussed in the first section: wealth decumulation rates, consumption volatility and annuitization rates.

Slower wealth decumulation

Unsurprisingly, an uncertain future liability stream increases precautionary savings motives (Deaton 1991; Carroll 1992; Hubbard, Skinner and Zeldes 1995). This desire to self-insure against risky future spending increases the motivation to preserve assets and slows the rate of wealth decumulation.

With this new model, individual, nonmarket risks suddenly appear very relevant to retirement planning. In Exhibit 5, we depict the average wealth decumulation under the model with an uncertain liability process for a retiree with initial wealth of $1 million ("Precautionary"), a process for a retiree who was very lucky and saw no required expenses ("Healthy"), a process for a retiree who was very unlucky and in the fifth percentile of the distribution and, finally, the naive decumulation rates from the canonical Yaari-style model discussed in Section 1 ("Naive").

Exhibit 5: Wealth decumulation with uncertain liabilities



Hypothetical example for illustrative purposes only. Source: Author's calculations

Consistent with the empirical evidence, health outcomes significantly affect retirees' remaining wealth. A healthy 85-year-old has approximately 45% more assets than an individual in the lowest fifth percentile of the cumulative distribution, which is still slightly higher than the remaining wealth predicted by the canonical model.

Volatile consumption: Joint targeting of income and wealth

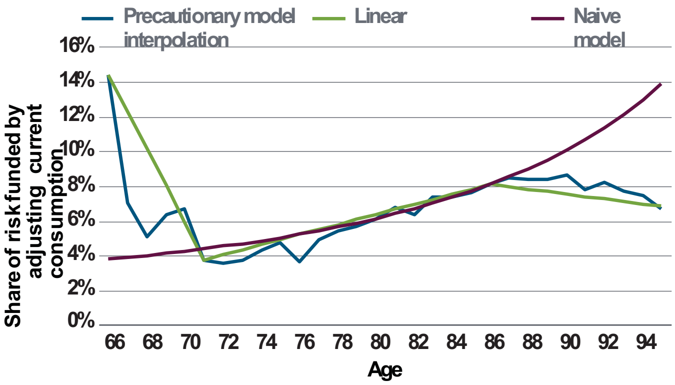
In this model, required spending can be funded in one of two ways: Either current spending can fall, or wealth can fall to meet the required healthcare shocks. The results suggest that retirees will raise and lower current discretionary spending to partially offset required outflows. Discretionary spending in retirement will be volatile, as this volatility helps preserve wealth for future needs. Early in retirement, uncertain required expenses are funded through larger reductions in discretionary spending than they are later in retirement. Only very late in retirement does the opposite relationship hold as retirees finally spend down their excess wealth. Exhibit 6 shows the fraction of uncertain expenses that are funded by changes in current discretionary consumption (the remainder is funded from wealth), by age. Early in retirement, as much as 15% of required expenses are paid for by immediate decreases in discretionary consumption. This share declines rapidly with age until age 70, when 4% of shocks are funded by changes in consumption and 96% from asset balances.

13 Brown and Finkelstein (2011)

14 Social Security Administration "Income of the Aged Chartbook," 2014.

15 For example, with the baseline parameterization, consumption guarantees at or below the federal poverty line lead to a net slowdown of wealth decumulation rates.

Exhibit 6: Funding uncertainty: Wealth versus discretionary consumption



Hypothetical example for illustrative purposes only. Source: Author's calculations

Discretionary consumption responses are estimated via linear regression on simulated shocks evaluated through the optimal consumption function. The age profile presented is smoothed using a piecewise linear transformation with knots at ages 70 and 85.

As a percentage, discretionary consumption is five times as responsive as wealth to required expenses at the beginning of retirement, when a 1% decline in wealth corresponds to a 5% decline in consumption. This falls over time until the elasticity of consumption and wealth converge after 10 to 15 years. A declining profile is the reverse of the effect of wealth shocks in a Yaari-style model,[16] in which the retiree would more significantly draw down their wealth early in retirement to stabilize lifelong income. Between the ages 70 and 85, the funding pattern nearly matches the canonical model, and late in retirement we see accumulated precautionary wealth increasingly used to help dampen the volatility in late-retirement discretionary consumption.

The patterns in discretionary consumption volatility suggest different roles for assets in early, middle and late retirement. Initially, preserving assets is paramount and spending is relatively responsive to ensure wealth can be maintained to fund a full retirement. As time passes, the assets are increasingly relied upon to finance late-retirement needs. Exhibit 6 suggests that assets are spent down differently depending on whether the retiree is in early, middle or late retirement.

Lower annuity purchases

Uncertain future expenses lead the retiree to tolerate more income volatility early in retirement in exchange for the preservation of future wealth. It is no surprise, then, that retirees in this model would place a lower value on a stable income stream. Higher, fixed payments are not useful if they deplete assets enough at the outset so that the retiree is unable to absorb potentially large future expense shocks. The behavioral literature explicitly supports this precautionary aversion to annuities: Beshears et al. (2014) document the top self-reported reasons for not purchasing an annuity as "[I] want to ensure I have enough income later in life," "[I] want flexibility in the timing of my spending," and "[I] may have a big (unanticipated) spending need." They also report that presenting annuities and highlighting the loss of "flexibility and control" of assets is associated with the greatest reduction in annuity purchases.

As shown in Exhibit 7, optimal annuity purchases are cut by 50%–100% relative to a Yaari-style model.[17] Based on these results, precautionary saving appears to play a part in the limited real-world appeal of annuities. Annuity purchases fall more with lower levels of initial wealth, as relatively larger unknown expenses provide a stronger desire to preserve assets when they are low.

Exhibit 7: Optimal annuity purchase with uncertain liabilities

	Life expectancy	Risk aversion	Discounting	Wealth ($000)	Optimal annuity share: Yaari model	Optimal annuity share: Uncertain liabilities
1	Male	4	2.50%	$1,000	100%	54%
2	Female	4	2.50%	$1,000	100%	44%
3	105	4	0.00%	$1,000	100%	36%
4	Male	2	2.50%	$1,000	65%	61%
5	Male	8	2.50%	$1,000	100%	16%
6	Male	4	2.50%	$210	100%	0%
7	Female	4	2.50%	$210	73%	0%

Hypothetical example for illustrative purposes only. The model outputs included here are not based on any particularized financial situation, or need, and nothing contained herein should be considered investment or retirement advice. Aura does not offer insurance guaranteed products or products that offer investments containing both securities and insurance features. The model is limited to analyzing the optimal annuitization rates. Investors should speak to their financial professional regarding the investment mix that may be right for them based on their financial situation and investment objectives. Source: Author's calculations.

Life expectancy data is from the Social Security Administration. Annuities are priced at a money's worth of 80%. Life expectancy of 105 represents a retiree who lives to age 105 with 100% probability. Annuity shares are in addition to a guaranteed Social Security payment equal to the statutory maximum for a full retirement age claimant.

Of particular note is the effect of life expectancy and risk tolerance. Absent spending requirements, a longer life expectancy for an individual should increase the appeal of a lifetime annuity. Here we see the opposite: The long-lived 105-year-old – who would normally be an ideal annuity customer – actually has a stronger desire to avoid annuities in a world with uncertain required spending. Although annuities provide an exceptionally high rate of return in this case, the retiree also runs a higher risk of larger expenses later in retirement that exceed the obtainable income stream from annuitization. Using the healthcare expense distribution in Exhibit 4, a 105-year life expectancy suggests an average lifetime out-of-pocket expense of nearly half a million dollars and a 95th percentile cost of over $750,000! The increased appeal of longevity protection is weighed against the increased desire to accumulate precautionary wealth. These results suggest that the second effect appears to dominate the first. Risk tolerance has a similar counterintuitive effect: Increases in risk aversion decrease the appeal of annuities, reversing the conclusion of typical retirement income models. With uncertain liabilities, longevity risk is no longer the retiree's primary concern.

We do not consider state-contingent annuities – contracts that pay more in certain states potentially unrelated to financial markets. These products are essentially nonexistent. Brown, Davidoff and Diamond (2005) explicitly mention incomplete markets (those without state-contingent annuities) combined with precautionary savings motives – exactly as we have in this model – as one in which annuitization may not be optimal.

CONCLUSION

Real-world retirees exhibit several behaviors that conflict with the predictions of canonical retirement income models. They appear overly averse to spending their accumulated wealth, their consumption is far too volatile, and they buy far too few annuities. With few exceptions, the response of the retirement industry to this behavior has been to try and persuade retirees to behave more like the model prescribes. This included designing drawdown strategies, facilitating annuity purchases and designing products to help mitigate volatile spending.

The focus on the retiree's assets ignores large risks that real-world retirees must manage. They face expenses that are not easy to forecast or anticipate. Even when these risks cannot be hedged by financial instruments, they have significant impacts on behavior. This is not theoretical: There is a large literature that shows health status and health expenses dramatically impact asset balances, portfolio allocations and total spending. Indeed, the desire to save for future healthcare expenses is a well-documented self-reported concern, particularly among the elderly.

This paper explicitly incorporates unknown required future expenses into the retirement planning problem by parameterizing a healthcare expense process based on real-world out-of-pocket expenses. Incorporating this feature appears to help close the gap between model-predicted and real-world behavior. This view can partially rationalize a series of otherwise puzzling behaviors, including slow drawdown of retirement wealth, excess consumption volatility and a lack of annuitization.

Financing an Uncertain Retirement Part II: Portfolio Construction

Executive Summary

- The risk of having insufficient savings or even running out of money in retirement is not accounted for in typical economic models even though it cannot be hedged in financial markets and may affect retirees' consumption and investment behavior.
- In this paper, we augment the traditional retirement model to account for unknown required future expenses that reflect actual retiree spending and investment behavior more closely.
- Our model predicts that retiree spending will exhibit some volatility as individuals seek to preserve their wealth and their optimal asset allocation should slowly de-risk over time.
- Instead, when future consumption is unknown and has increasing uncertainty over time, rolling shorter-term investment strategies, such as a bond ladder, may be more suited to a retiree's needs.

Introduction

This paper is the second in a two-part series that explores the implications of uncertain future liabilities for savings and investment in retirement. The first paper, "Financing an Uncertain Retirement Part I: Spending Strategies," extends canonical retirement models to include uncertain required future expenses, such as out-of-pocket healthcare spending. The resulting model is a straightforward application of the common fact that people do not know how much money they will need in retirement and fear that they might run out of assets. Uncertain future expenses lead to behaviors that help address several otherwise-puzzling data among retirees, including slow wealth decumulation rates, low annuitization rates and volatile consumption in retirement.

Required but uncertain future expenses have material asset allocation implications as well. Even when the expense uncertainty cannot be easily hedged in the financial markets, its presence has investment implications. This paper makes three contributions to the literature on retirement savings: First, it presents and solves a model with uncertain future spending needs whose predictions more closely reflect real-world consumption and asset allocation behavior; second, it characterizes the subtle and nonmonotonic relationships among wealth, spending and asset allocation in the presence of unknown future expenses; and, finally, it describes potential ways to design investment portfolios and rebalancing strategies to account for uncertain future spending needs.

REQUIRED EXPENSES: MOTIVATION AND PARAMETERIZATION

A full 49% of American adults cite running out of money as their top retirement concern.[1] Unfortunately, common economic models do not accommodate this fear. In most economic models, a rational retiree would simply spend less as their wealth fell. That is, these models view all spending as a choice rather than a requirement. Instead, we propose to explicitly incorporate these concerns, as individuals face unknown required future expenses at unknown times throughout their retirement. There are many potential sources of these shocks: unexpected home repairs or maintenance, potential tax/policy changes or even uncertainty over future preferences. Of course, the most immediate example of this type of spending is one of the largest and fastest-growing expenses for retirees: healthcare.

Healthcare represents large, very volatile expenses throughout retirement. Once individuals reach age 70, their households will, on average, incur over $122,000 in out-of-pocket medical spending over the remainder of their lives, while households in the highest percentile of expenditures will incur over $600,000 in medical expenses over their remaining lives (Jones et al. 2018). Unsurprisingly, 45% of adults are concerned they will not be able to afford healthcare when they retire.[2] Required expenses are designed to roughly follow the distribution of out-of-pocket medical expenses by age in Jones et al. (2018) and are summarized in Exhibit 1.

Exhibit 1:Distribution of modeled out-of-pocket healthcare expenses ($000)

Age	65	75	85	95	Lifetime
μ	0.41	0.98	1.37	1.65	
σ	1.34	1.39	1.46	1.51	
Mean	3.1	5.9	9.3	13.2	126
Median	1.5	2.7	3.9	5.2	116
75th percentile	3.6	6.7	10.3	14.2	150
90th percentile	7.8	14.9	23.9	34.0	193
95th percentile	12.1	23.5	38.5	55.8	222

Source: Author's calculations.

Percentiles are expressed in thousands. Lifetime values reflect the Social Security Administration's forward-looking survival rates for a 65-year-old male. The simulation is drawn from a lognormal distribution parameterized by mean μ and variance σ^2.

Unsurprisingly, both the expectation and the variance of out-of-pocket healthcare expenses increase with age. For convenience, healthcare spending is assumed to be independent through time and uncorrelated with mortality. Despite its simplicity, this process recovers a lifetime distribution of out-of-pocket costs near to, or slightly conservative when compared with, their true values (Jones et al. 2018 and Fronstin et al. 2014).

This paper complements the sizable empirical research that demonstrates the wealth and allocation consequences of healthcare expenses (see De Nardi, French and Jones 2010; Coile and Milligan 2009; Rosen and Wu 2004; Poterba, Venti and Wise 2011; Poterba, Venti and Wise 2017), and spending behavior in retirement (Blanchett 2013, Banerjee 2015), by presenting and solving a model of joint expenditure and asset allocation in the presence of uncertain required liabilities. In contrast to typical models, a framework with uncertain required liabilities predicts consumption and asset allocation behavior that far more closely reflects real-world patterns, in our view.

THE MODEL

A retiree chooses a sequence of in-retirement consumption $(\{c_t\}_{t=0}^{T})$ and asset allocations $(\{\varphi_t\}_{t=0}^{T})$ to maximize utility subject to uncertain realizations of required expenses each year $(\{h_t\}_{t=0}^{T})$. Utility is defined over consumption only, exponentially discounted (β) and weighted by mortality (π_t).

$$\max_{\{c_t\}_{t=1}^{T},\{\varphi_t\}_{t=1}^{T}} \sum_t \beta^t \pi_t E[U(c_t - h_t)].$$

Utility is maximized subject to the period-by-period budget constraint and evolution of the retiree's invested wealth based on stochastic returns to the risky asset μ_t:

$$c_t \le W_t + SS_t - h_t$$

$$W_{t+1} = W_t(\varphi_t \mu_t + 1) - c_t.$$

Here households may truly run out of money with required expenses. If this occurs, they are provided a minimum consumption guarantee[3] set equal to c_t:

$$h_t > W_t + SS,\ \{c_{t+s} - h_{t+s}\}_{s=1}^{T} = \{c_{t+s}\}_{s=1}^{T},$$

$$\{W_{t+s}\}_{s=1}^{T} = 0, \{\varphi_{t+s}\}_{s=1}^{T} = 0.$$

1 Edleson. "Almost Half of Americans Fear Running Out of Money in Retirement" (2019)

2 Institute for Healthcare Policy and Innovation, University of Michigan, 2019

3 Brown and Finkelstein (2011)

Of course, with any type of guarantee, it is possible to have a rapid divesting of all assets be the dominant strategy, as this maximizes the value of the guarantee for the remaining years. This is not a widely observed strategy among retirees (nor does it reflect typical advice from financial advisors), suggesting that real-world welfare guarantees represent a rather low level of consumption. As in Part 1 of this series, the consumption guarantee is set to the fifth percentile of income for retirees in the U.S.[4] For simplicity, portfolios are constructed out of two stylized assets: a “risk-free” security that pays a zero rate of return in every state of the world, and a risky investment that pays a positive expected return that is not guaranteed. Here the risky investment is intended to represent equities: The expected return is 3% over the risk-free asset, with an annual volatility of 16%:

$$\mu_t \sim N(3\%, 2.56\%)$$

Required expenses, h_t, are as specified in the previous section. As in the first paper in this series, preferences are expressed as constant relative risk aversion ($U(c) = \frac{c^{1-\rho}}{1-\rho}$), with a coefficient of relative risk aversion equal to 4. Individuals discount future consumption at a rate of 2.5% per year ($\beta = 0.975$), and survival rates are those for a 65-year-old male, truncated at age 105.[5]

Optimal consumption and asset allocation decisions are determined by the retiree’s age, level of wealth and required expenses:

$$\varphi_t^* = \varphi(t, W_t, h_t)$$

$$c_t^* = c(t, W_t, h_t)$$

The solution to this problem is a set of state-dependent functions (typically called controls) that depend on the retiree’s age and wealth, and the realized expense requirements. The solution to this model will be presented in three stages with gradually increasing complexity.

Case 1: No required expenses, no Social Security

First, we consider the solution to a more traditional model, without required consumption or Social Security. This case is well known but somewhat unique. Here the optimal asset allocation function can be solved for in closed form (similar to Merton 1969 and 1973). The investor will hold a constant equity allocation throughout equal to

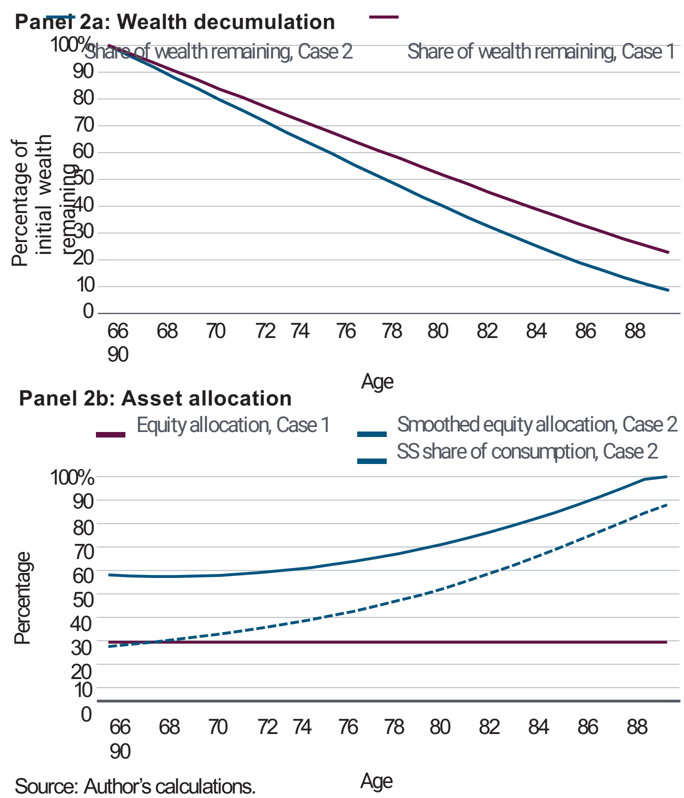
$$29.2\% \left(\varphi^* = \frac{\mu - r}{\rho\sigma^2} = \frac{0.03}{4(0.16)^2} = 29.2\%\right).$$

Subsequent cases in this paper cannot be solved analytically and require numerical methods.

Case 2: Including Social Security

The presence of guaranteed period-by-period income materially changes the incentives facing the retiree. In this first extension, Social Security benefits are provided to the retiree at their 2019 maximum of $34,332 per year. These benefits effectively make the retiree much wealthier: In addition to their financial wealth of $1 million, they now have a substantial stream of guaranteed income.

Exhibit 2: Decumulation rates and asset allocation, Case 1 and Case 2



Source: Author’s calculations.

Smoothed allocation represents a quadratic fit of the numerical solutions to remove noise.

4 This seems realistic: Individuals receive long-term-care coverage through Medicaid with very few out-of-pocket costs only after paying a deductible of essentially all their remaining assets. Reasonable changes in this guarantee affect the magnitude of the findings presented in this paper, but not their existence. For example, any guarantees at or below the poverty line would lead to slower wealth drawdown rates than those found in the model without uncertain required consumption.

5 Social Security Administration Actuarial Life Table, 2010. In this paper, analysis is focused on behavior until age 90 to ensure that the results are not driven by boundary issues induced by the numerical solution method, such as the known terminal age and bounded support of the health expense shocks.

This effectively provides them a minimum level of consumption, which tends to accelerate the drawdown rate of their wealth, as shown in Panel 2a of Exhibit 2. At age 82, the retiree in Case 2 (with Social Security) has 25% less financial wealth than the retiree in Case 1 (without Social Security), and this declines to less than 50% at age 88. The asset allocation consequences in Panel 2b are similarly pronounced. Intuitively, Social Security represents a large part of the Case 2 investor's wealth. Invested assets play a smaller role in financing spending with Social Security, so the net effect of asset volatility on total consumption falls in the presence of guaranteed income payments. Panel 2b of Exhibit 2 shows that with these parameters roughly 25%–30% of initial retirement consumption is financed by Social Security payments. Over time, total spending falls and the fraction of consumption sourced from Social Security increases, passing 50% at age 80 and reaching 80% by the retiree's 90th birthday. The volatility of financial wealth has a diminishing impact on the volatility of total consumption over time. Accordingly, the optimal portfolio holds an increasing allocation to equities as the retiree ages, until it is essentially fully invested in risk assets from age 90 onward.

The consumption capital asset pricing model (CCAPM; Rubinstein 1976, Lucas 1978, Breeden 1979) provides another way to understand the relationship between Social Security and optimal asset allocation. In the CCAPM, assets less correlated with consumption are more attractive investments. As the retiree ages, their total consumption is composed increasingly of Social Security and is sourced less from their invested assets. As a result, risky investments become increasingly attractive.[6]

The upward-sloping allocation to risk assets is entirely driven by the changing relative importance of Social Security to overall consumption.

Needless to say, increasing investment risk with age is not behavior exhibited by retirees. Averaging across the 1989–2016 data, the Federal Reserve's Survey of Consumer Finances reports the share of financial assets in equities drops from 55% for households ages 55–64 to 48% for households ages 75 or more.[7]

The contradiction between real-world actual allocations in retirement (which modestly decline) and upward-sloping "optimal" equity allocations is well known. In a model very similar to what we saw in Cases 1 and 2, Delorme (2015) and Daverman and O'Hara (2015) show the role that the lack of wage income and the presence of pension income, such as Social Security, play in driving the optimal risk allocations upward. An upward-sloping allocation to risk assets in retirement is a consequence of combining typical retirement income models with the presence of lifetime income streams, such as Social Security. Such an investment profile is not seen in the data, and extremely risky portfolios for the very aged are not often recommended by financial professionals.[8] In contrast, uncertain future expenses lead to more realistic spending and investment behavior, even with guaranteed income and constant risk preferences.

Case 3: Incorporating uncertain required expenses

Next we incorporate spending requirements into the model that follow the distribution shown in Exhibit 1. Optimal consumption and allocation behavior,

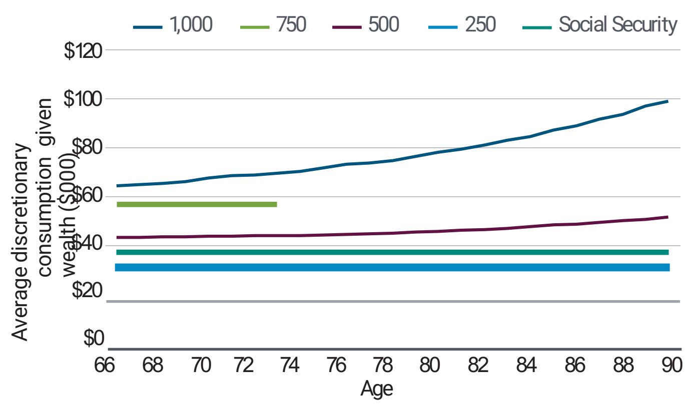
$$c_t^* = c(t, W_t, h_t) \text{ and } \varphi_t^* = \varphi(t, W_t, h_t),$$

are explored in sequence.

OPTIMAL CONSUMPTION BEHAVIOR

Exhibit 3 shows optimal discretionary consumption behavior. Additional wealth increases optimal consumption at any age. Similarly, any given level of wealth later in life leads to higher consumption, as the retiree is effectively "richer," with fewer expected years of spending to finance. Wealth preservation behavior also is visible in this exhibit: With low enough asset balances, discretionary spending eventually falls below guaranteed Social Security income as retirees decrease consumption in an attempt to increase asset balances.

Exhibit 3: Discretionary consumption by age and wealth



Source: Author's calculations.

Values represent the optimal amount of consumption in excess of required expenses by age for a given level of wealth at that age.

6 This is a more subtle benefit of higher guaranteed income, such as that from deferring Social Security benefits: Total consumption becomes less correlated to the markets, allowing for higher comfortable allocations to risk assets and correspondingly higher expected returns.

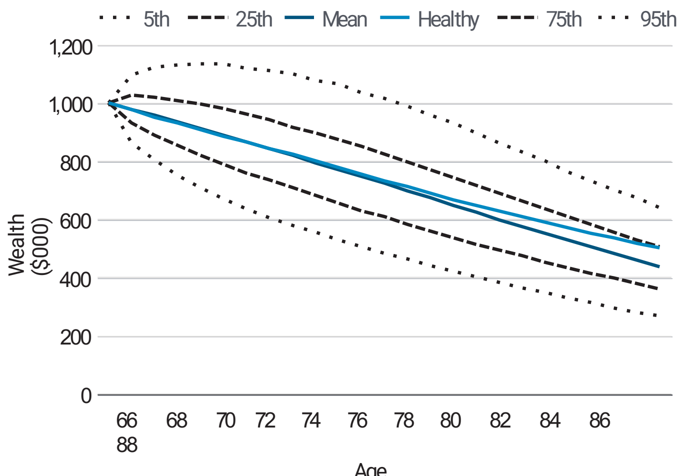
7 Federal Reserve Survey of Consumer Finances. Average from 1989–2016 surveys.

8 Pfau and Kitces (2014) are an exception. Some of their work advocates for an upward-sloping-in-retirement glide path, based on the survival rates of certain spending rules.

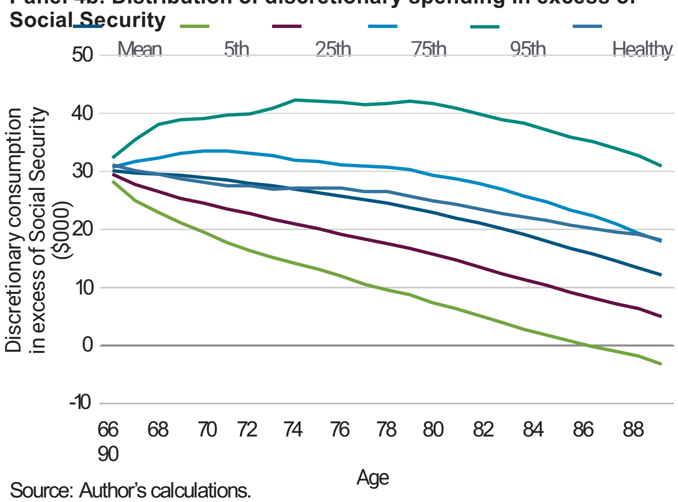
For higher wealth levels, optimal spending (and thus wealth decumulation) rates in this model are generally positive, so that a well-funded retiree expects to slowly spend down their assets over time. Of course, the realized rate is subject to uncertainty in asset returns and expense requirements: Uncertain financial markets and unknown future spending needs combine to create a distribution of spending and wealth that slowly declines over time. Exhibit 4 shows percentiles of the distribution of wealth (Panel 4a) and discretionary consumption financed by wealth (Panel 4b), as well as the path followed by a (very lucky) healthy retiree who never experiences required health expenses and whose investments always receive their expected returns.

Exhibit 4: Distribution of wealth and self-financed discretionary consumption over time

Panel 4a: Distribution of wealth



Panel 4b: Distribution of discretionary spending in excess of Social Security



Source: Author's calculations.

Values represent the percentiles of the distributions of simulated wealth and discretionary consumption financed by wealth. "Healthy" individuals experience no required health expenses and always receive the expected equity returns with zero volatility; these values are smoothed with a moving average to remove artifacts from the grid-based solution.

There are two immediate conclusions. First, as stated above, both wealth and discretionary consumption tend to decline over time; second, they are both quite volatile. The combined impacts of health expenses and asset market shocks on wealth are only partially absorbed by movements in discretionary spending. Both of these predictions agree with the data on wealth and spending patterns in retirement. Excluding healthcare spending, real consumption declines as households age, and households with larger health expenses tend to have smaller asset balances (Rosen and Wu 2004, Ebrahimi 2019, Consumer Expenditure Survey).

OPTIMAL ASSET ALLOCATION

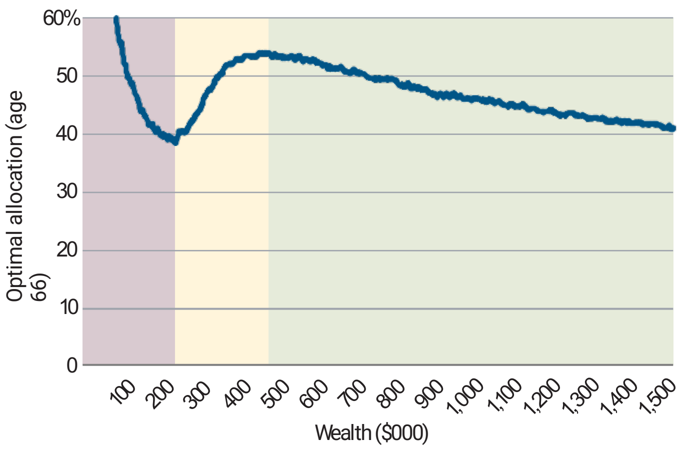
Optimal asset allocation behavior with required expenses $(\varphi_t^* = \varphi(t, W_t, h_t)$ is more subtle. Unlike consumption, optimal risk-taking behavior need not have a monotonic relationship with wealth, particularly when current assets are very near the required amount of future funding. If the individual has enough assets, extra risk will increase the probability of a sufficiently bad outcome and decrease the chances that they can meet their funding requirements. If, however, they have too little wealth, extra risk increases the chance of a sufficiently positive outcome to meet their funding requirements. This may seem counterintuitive, but this relationship is surprisingly common. For example, increasingly risky strategies are often chosen in sports when a loss is otherwise assured: Pulling a goalie in ice hockey or immediately fouling in basketball are high risk strategies with very high variance (and negative expected returns). These sorts of strategies are typically seen from the losing team near the end of the game, when a loss is almost certain, but never implemented by a team in the lead. Similarly, when wealth levels fall below anticipated future spending needs, risk-taking becomes increasingly attractive, as it provides a greater chance of sufficient funding in future periods.

9 The unconditional wealth distribution initially widens, then narrows later in retirement as retirees adjust their period-by-period consumption and attempt to maintain a level of future wealth that allows for continued self-insurance against future risks. This is seen most clearly with the "healthy" retiree, whose wealth and consumption do not materially deviate from the mean trajectory until 10 to 15 years into retirement. Only at this point does the healthy retiree's consumption increase, as their excess wealth is finally consumed rather than saved. Consistent with this, the distribution of wealth in Exhibit 4 begins to narrow at age 75, even though the volatility of healthcare expenses grows rapidly well beyond this point.

This pattern is borne out in the model's results, as shown in Exhibit 5. When wealth is high relative to future needs (in the green region), decreases in wealth slowly increase the optimal equity allocation, as the risk of underfunding is low and Social Security plays a larger role in total consumption, as we saw in Case 2. (The 95th percentile lifetime required consumption is

$222,000, per Exhibit 1.)

Exhibit 5: Nonmonotone wealth allocation relationship early in retirement

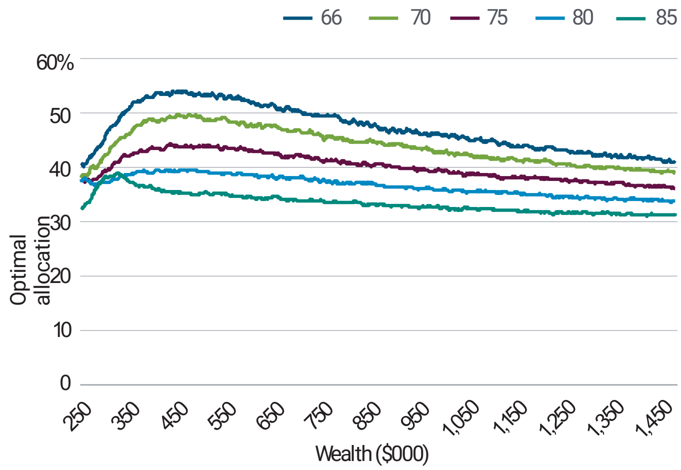


Source: Author's calculations.

Values are smoothed with a moving average to remove artifacts from the grid-based solution.

However, as wealth declines further, funding risk gradually increases. Eventually, as wealth nears the amount necessary to fund future required expenses, optimal allocations begin to fall. In this range – approximately $200,000 to $450,000 for a 66-year-old and denoted in yellow in Exhibit 5 – the increased risk of failure from higher portfolio volatility outweighs the Social Security effect, and the net appeal of risky investment is diminished. Finally, for wealth levels that fall below future required expenses – less than $250,000 and marked in red in the exhibit – additional risk lowers the risk of failure in the future and is increasingly attractive. The magnitude and timing of this relationship evolve over time, as older retirees have fewer periods before they face the potential of large required expenses. In Exhibit 6, we show the optimal equity allocation for ages 66, 70, 75, 80 and 85 for retirees who appear "funded," with wealth beyond $250,000.

Exhibit 6: Wealth allocation relationship by age for funded retirees



Source: Author's calculations.

Values are smoothed with a moving average to remove artifacts from the grid-based solution.

As retirees age and spend their assets, they move down between the curves in Exhibit 6. The ultimate relationship between asset allocation and age is a function of how quickly wealth is spent down (how rapidly the individual moves toward the origin). There is a wide range of potential dynamics here: Consider a household with wealth levels near the median of $250,000.[10] These households likely have sufficient wealth to cover their future medical expenses, though they are relatively close to the funding boundary (approximately $250,000 to $450,000, as in Exhibit 5), and declines in wealth lower the appeal of equities. These results suggest that such households will hold approximately 40% in equities. Moreover, as Exhibit 3 shows, these households may actually consume slightly less than their Social Security payments to preserve their savings for unknown future expenses. If markets perform well, the extra returns plus these extra savings will increase the households' wealth and drive up their optimal allocation. A market downturn will have the opposite effect: lowering their wealth, making equity risk less attractive at the margin and leading to a reduction in risk asset exposures.

That is, a retiree in this region will increase exposure to equities after a rally and reduce it after a drawdown – exactly the sort of "buy high, sell low" behavior among retail investors that is so often denounced. Here, though, such trading would likely be optimal, given the nature of the risks they face.

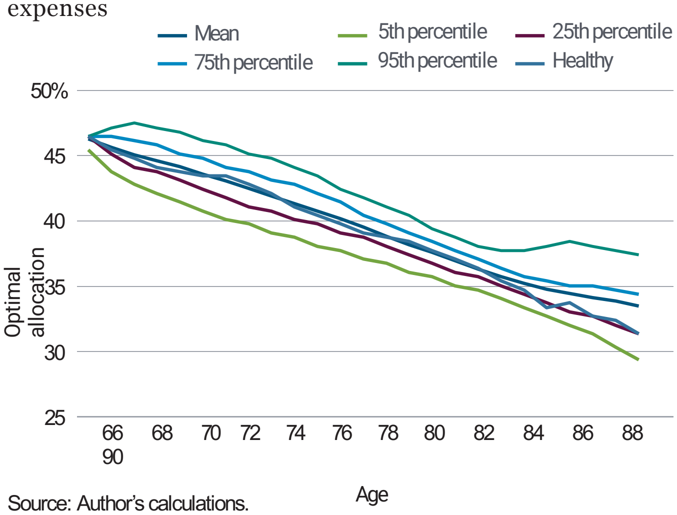
10 Federal Reserve Board, 2016 Survey of Consumer Finances. Median net worth for 65–74-years old and 75 years old and over households is $223,400 and $264,800, respectively.

These results suggest that, in addition to lowering current and future consumption to preserve wealth, as discussed in the first part of this series (Klein 2020), the reduction in wealth from required expenses will lead to less risky asset allocations for many moderate net worth households or those with higher anticipated healthcare spending needs, as documented in Rosen and Wu (2004) and Poterba, Venti and Wise (2017).

A DE-RISKING ASSET ALLOCATION THROUGH TIME

With $1 million in initial financial wealth, the retiree's joint consumption and wealth decumulation behavior is such that the optimal equity allocation tends to decline slowly with age. Compared with the volatility of discretionary consumption (in Exhibit 4), the range of acceptable asset allocations is rather narrow: The difference between the lowest and highest is only 5 to 10 percentage points throughout the early and middle retirement years as shown in Exhibit 7.

Exhibit 7: In-retirement glide path with uncertain expenses



Source: Author's calculations.

Values are smoothed with a moving average to remove artifacts induced from the grid-based solution. "Healthy" individuals experience no required health expenses and always receive the expected equity returns with zero volatility.

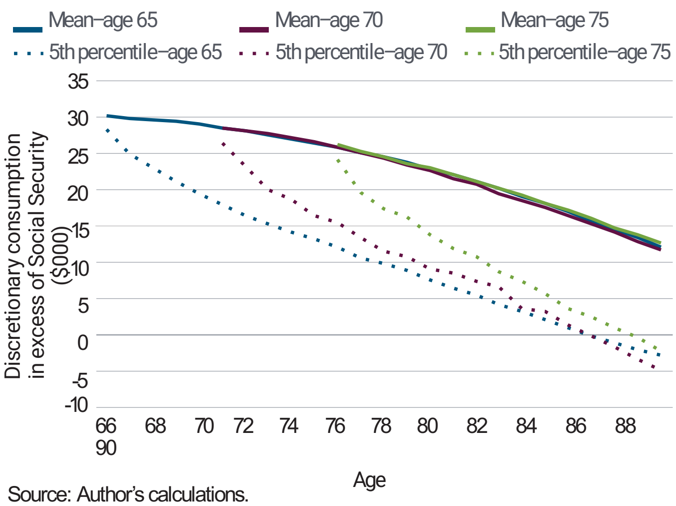
Volatile consumption, annuities and rolling ladder strategies

Uncertain required spending no longer makes constant consumption the most desirable outcome. Instead, spending behavior will rise and fall with markets and individual circumstances. Such variation is sensible but immediately limits the appeal of steady income streams. In a model that excludes uncertain required spending, annuities are nearly ideal investment products. Here they can still be attractive, but their appeal may be more as an investment that harvests an increasing survival premium than as a way to guarantee fixed total consumption for many years. If actuarially fair, one-period-ahead annuities would still be ideal investments (Brown et al. 2005, Peng and Warshawsky 2010), as the guaranteed return is inversely proportional to ever-declining survival rates. Of course, annuities are not actuarially fair. Using the survival rates in this paper, a one-period annuity's return will exceed a 3% equity risk premium once the individual reaches age 73, but it cannot overcome an 85% money's worth (the average value from Brown et al. 2000) until the retiree reaches age 90. An annuity targeted toward harvesting high survival premia that spreads the money's worth costs over several years, such as a deferred, or longevity, annuity, seems appealing because of the high return. However, this return must be weighed against desired future consumption, which is steadily falling and may not always exceed other sources of income. We can see this in Panel 4b of Exhibit 4: Even with $1 million in starting wealth, in the very worst scenarios consumption need not exceed Social Security benefits 20 years into retirement.

A retiree in this model expects to enjoy substantial consumption beyond their Social Security benefits, on average, particularly early in retirement. As the retiree ages, they face a distribution of possible future total consumption financed from the investment portfolio (as in Exhibit 4) such that in the worst cases they will have no discretionary spending beyond their Social Security benefits. The size and shape of the distribution of their future spending are a function of the distribution of required future expenses, and their wealth – itself determined by realized market returns, Social Security payments and the history of their overall portfolio allocation. Assuming the term premium is not extremely negative (indeed, it is usually positive), a risk-averse retiree would at least want to ensure the lower bound of the distribution of future consumption on a rolling basis.

Exhibit 8 shows the distribution of future consumption for retirees aged 65, 70 and 75, conditional on their remaining wealth being within $10,000 of the mean value expected at that age.

Exhibit 8: Conditional distribution of future consumption financed by wealth, ages 65, 70 and 75



Source: Author's calculations.

Values are smoothed with a moving average to remove artifacts induced from the grid-based solution. For the age 70 and 75 values, the sample is restricted to the 2%–3% of paths with simulated wealth at the indicated age within $10,000 of the mean value.

At age 66, the fifth percentile of consumption begins at nearly $30,000 and declines until it reaches zero near age 85, even though at this age expected consumption is $15,000 to $20,000 higher. Once the retiree reaches age 70, the forward-looking distribution widens more rapidly. Conditional on the retiree having close to the mean level of wealth at age 70, the fifth percentile again begins near the mean, but it declines to zero at nearly the same age as in the unconditional distribution. This suggests that rolling, shorter-term, partial consumption immunization strategies are preferred in a world with uncertain future expenses.[11] Rather than support constant spending for life, the retiree would want assets that generated payments equal to conservative percentiles of their anticipated spending. As an example, a rolling, intermediate-term bond ladder may be a more attractive way to support decreasing future spending than would an immediate annuity. The height and length of the ladder would evolve as the investor moved through retirement and uncertainty (in both the markets and their own expense needs) was resolved.

Finally, Exhibit 8 shows that, in addition to helping to protect different amounts of real income each year, investment strategies should slowly shorten their maturity over time. As the retiree ages, they steadily approach the possibility of immediate, large required expenses. An increased potential need for short- term cash reduces the appeal of investments that protect consumption well into the future. In the exhibit, the tail percentiles steepen as the retiree ages. As a result, investments following this distribution would shorten maturities over time.

Using the positive portion of the fifth percentile in Exhibit 8 as a guide, weighted average spending moves from 7.25 years in the future at age 66 to 6.0 at age 70 and 4.7 years at age 75.

11 Of course, the desired amount of future consumption to protect is a function of capital market assumptions, particularly the term premium and the covariance between investable assets and future consumption needs.

CONCLUSION

A top self-reported concern is that individuals may run out of money in their retirement or otherwise be unable to afford their expenses. Typical economic models do not easily permit these sorts of statements. In this paper, the traditional retirement model is augmented with unknown required future expenses. These expenses can be thought of as an approximation to uncertainty over future preferences or random future states in which the marginal utility of consumption is very high, such as required repairs, taxes or – the example explored in this paper – out-of-pocket healthcare spending. Though these idiosyncratic risks cannot be hedged in the financial markets, this does not mean that they will not affect consumption and investment behavior.

With the inclusion of unknown required future liabilities, model predictions reflect actual retiree behavior much more closely. The first paper in this series found that wealth would be spent much more slowly and consumption would be more volatile, relative to the canonical model, to preserve assets for risks that might be faced later in retirement. Similarly, annuities would be less attractive, particularly for those with higher risk aversion and longer life expectancies – exactly those who would most enjoy annuities under traditional models.

This paper extends the framework to investigate asset allocation. In traditional models, the presence of Social Security or other guaranteed pension benefits typically results in asset allocations that continually increase their portfolios' risk as retirees age – yet another result not seen in the data. With uncertain liabilities, the optimal asset allocation is determined by whether the retiree has enough wealth relative to the distribution of future risks.

Including uncertain required future expenses generates several predictions that more closely reflect real-world investment behavior:

- Non-healthcare-related spending should fall as the retiree ages, while spending on healthcare should increase.
- Spending in retirement should display some volatility to help preserve assets and protect against potential future risks.
- For retirees with enough wealth, the optimal asset allocation should slowly de-risk over time.
- Annuities or deferred annuities may not be attractive investments, even in the face of large positive survival premia.
- Instead, when future consumption is increasingly uncertain over time, rolling, shorter-term investment strategies that gradually shorten over time, such as a bond ladder, may be more suited to the retiree's needs.

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